Eur. Phys. J. A 3, 377–380 (1998)

THE EUROPEAN PHYSICAL JOURNAL A

Contribution of the σ meson exchange to pionic double charge exchange reaction

Xiao-fu Lü[1,5], B.S. Zou[2], Hui-ming Ding[3,5], Yu-xin Liu[4,5,a], En-guang Zhao[5]

[1] Department of Physics, Sichuan University, Chengdu 610064, China
[2] Queen Mary and Westfield College, London E1, 4NS, UK
[3] Department of Physics, Jilin University, Changchun 130023, China
[4] Department of Physics, Peking University, Beijing 100871, China
[5] Institute of Theoretical Physics, Academia Sinica, P. O. Box 2735, Beijing 100080, China

Received: 5 August 1998
Communicated by V.V. Anisovich

Abstract. The contribution of the σ meson exchange to the pionic double charge exchange (DCX) reaction is investigated. A concrete calculation on the forward excitation function of the low energy DCX reaction $^{14}C(\pi^+, \pi^-)^{14}O$ has been performed. It shows that the contribution of the σ meson exchange can reproduce well the resonance like excitation function of low energy DCX reaction.

PACS. 25.80.Gn Pion charge-exchange reactions – 13.75.Gx Pion-baryon interactions

Pion-nucleus double charge exchange (DCX) reaction involves at least two nucleons and therefore is an ideal place for studying N-N correlation. It has then achieved a great deal successes in both experimental and theoretical investigations(see for example [1,2]). However, it is still an open question to explain the resonance like excitation function at low energies around $T_\pi = 50$ MeV. On one hand, it is believed to come from the dibaryon (d') resonance [3]. On the other hand, with conventional mechanisms such as the improved coupled channel scattering theory with many low-lying state effect being included[4], the scheme with pion distortion being treated sophisticatedly[5] and the true pion absorption-emission mechanism[6], it can also be described. In this paper we will propose an alternative scheme to describe the excitation function of low energy DCX reaction.

Considering the meson exchange theory for N-N interaction, one knows that the pion exchange gives the largest range force; the $\pi - \pi$ S-wave (σ) exchange gives the second largest range force; then the ρ exchange and so on. It has been shown that including σ exchange is crucial for reproducing the medium range attractive potential for N-N interaction[7] and for reproducing the cross section of $pp \to pp\pi^0$ near threshold[8]. For DCX reactions, the contribution of the ρ exchange has been investigated in the same framework as for the π exchange[9] and found that it is not negligible for T_π around 50 MeV. However there has not yet been any report on the contribution of σ exchange. The reason for this may be that the the σ resonance was not regarded as an established resonance at that time. In the last few years, as the knowledge on the low energy $\pi - \pi$ S-wave resonance has been much improved[10, 11], the existence of a broad σ resonance (*i.e.*, $f_0(400 - 1200)$ in the PDG booklet) has been well established[11]. Therefore it is necessary to investigate the contribution of the σ meson exchange for DCX. Due to the fact that the σ exchange between two nucleons contributes an attractive force, the understanding on the contribution of σ exchange is very important for identifying other short range contributions such as the six-quark clusters[12, 13] and dibaryon resonance[3]. We will then investigate the contribution of σ exchange to the pionic double charge exchange reaction and discuss the excitation function of the DCX process $^{14}C(\pi^+, \pi^-)^{14}O$.

It has been shown that the fixed scattering center technique[14] is quite powerful in describing the pion-nuclear reactions (see for example [9] and references therein). Under the second order approximation of scattering theory with fixed scattering center, the position of every nucleon in the target nucleus is fixed in the DCX process. Let $\vec{r_1}$, $\vec{r_2}$ denote the coordinate of the two nucleons in the two sequential scattering processes, the effective reaction amplitude operator contributed by exchanging a σ meson can be written as

$$\hat{F}(\vec{k_f}, \vec{k_i}, \vec{r_2}, \vec{r_1}) = -\frac{1}{2\pi^2} e^{-i\vec{k_f}\cdot\vec{r_2}} \times \int d^3q f_2(\vec{k_f}, \vec{q}) g_\sigma(\vec{q}, \vec{r}) f_1(\vec{q}, \vec{k_i}) e^{i\vec{k_i}\cdot\vec{r_1}}, \quad (1)$$

[a] corresponding author

where $\vec{r}=\vec{r_2}-\vec{r_1}$, $\vec{k_i}$ and $\vec{k_f}$ are the incoming and outgoing momenta of the pion, rescpetively. The propagator g_σ of the σ meson is

$$g_\sigma(\vec{q},\vec{r})=\frac{e^{i\vec{q}\cdot\vec{r}}}{\vec{k_0}^2-\vec{q}^2+i\varepsilon}, \tag{2}$$

where k_0 is the on-shell momentum of σ meson. The $f_1(\vec{q},\vec{k_i})$ and $f_2(\vec{k_f},\vec{q})$ are the scattering amplitudes of the process $\pi^+ n\to p\sigma$ and $\sigma n\to p\pi^-$ respectively. When the following $\pi-N$ pseudo-vector coupling and $\sigma-N$ scalar coupling Lagrangians are considered

$$L_{\pi NN}=-g_{\pi NN}\bar{\Psi}_N\gamma_5(i\vec{\tau}\cdot\vec{\pi})\Psi, \tag{3}$$

$$L_{\sigma NN}=-g_{\sigma NN}\bar{\Psi}\Phi\Psi, \tag{4}$$

the scattering amplitudes in experimental frame can be given as

$$f_2(\vec{k_f},\vec{q})=\frac{\sqrt{2}}{4\pi}g_{\pi NN}g_{\sigma NN}\frac{1}{\omega_{k_f}}\frac{\vec{\sigma_2}\cdot\vec{k_f}}{2m_N}, \tag{5}$$

$$f_1(\vec{q},\vec{k_i})=\frac{\sqrt{2}}{4\pi}g_{\pi NN}g_{\sigma NN}\frac{1}{\omega_{k_i}}\frac{\vec{\sigma_1}\cdot\vec{k_i}}{2m_N}. \tag{6}$$

Accomplishing the angular integration in (1) with (2), (5) and (6) being included, the effective reaction amplitude operator can be simplified as

$$\hat{F}(\vec{k_f},\vec{k_i},\vec{r_2},\vec{r_1})=e^{-i\vec{k_f}\cdot\vec{R}}\hat{D}(\vec{k_f},\vec{k_i},\vec{r})e^{i\vec{k_i}\cdot\vec{R}}, \tag{7}$$

where $\vec{R}=\frac{1}{2}(\vec{r_1}+\vec{r_2})$ is the coordinate of the center-of-mass of the two nucleons and the contribution of the relative motion is

$$\hat{D}(\vec{k_f},\vec{k_i},\vec{r})=Ce^{-i\vec{k}\cdot\vec{r}}\vec{\sigma_2}\cdot\vec{e_f}\vec{\sigma_1}\cdot\vec{e_i}H(r), \tag{8}$$

where $\vec{K}=\vec{k_f}-\vec{k_i}$, $\vec{e_i}=\frac{\vec{k_i}}{k_i}$, $\vec{e_f}=\frac{\vec{k_f}}{k_f}$ and

$$C=-\frac{g^2_{\pi NN}g^2_{\sigma NN}}{16\pi^3m^3_N\omega_{k_i}\omega_{k_f}}k_fk_ik_0^2, \tag{9}$$

$$H(r)=\frac{m_N}{k_0^2}\int q^2dq\frac{1}{k_0^2-q^2+i\varepsilon}j_0(qr). \tag{10}$$

Denoting $|(n_1l_1j_1,n_2l_2j_2)J^+\rangle$, $|(n_3l_3j_3,n_4l_4j_3)J^+\rangle$ as the wave functions of the initial and final state with the two valence nucleons respectively and $\psi_i(\pi^+)$, $\psi_f(\pi^-)$ as that of the incoming and outgoing pion, the reaction amplitude can be given as

$$F(\vec{k_f},\vec{k_i})=\Big\langle(n_3l_3j_3,n_4l_4j_4)J^+\psi_f(\pi^-)\Big|\hat{D}(\vec{k_f},\vec{k_i},\vec{r},)\Big|(n_1l_1j_1,n_2l_2j_2)J^+\psi_i(\pi^+)\Big\rangle. \tag{11}$$

With the wave function of the pion being taken as the solution of the Klein-Gorden equation

$$(\nabla^2+U^{(\pm)})\psi_{\pi^\pm}(\vec{R})=k_0^2\psi_{\pi^\pm}(\vec{R}), \tag{12}$$

where $U^{(\pm)}$ is the optical potential of $\pi^\pm$-nucleus elastic scattering, the distortion effect of the external pion, which has been shown to play an important role in the DCX process[5, 6], is taken into account.

After a tedious derivation, we get the reaction amplitude as

$$\begin{aligned}F(\vec{k_f},\vec{k_i})=&\sum_{\lambda S}\begin{bmatrix}l_1 & \frac{1}{2} & j_1\\ l_2 & \frac{1}{2} & j_2\\ \lambda & S & J\end{bmatrix}\sum_{nlNL}M_\lambda(nlNL,n_1l_1n_2l_2)\\ &\times\sum_{\lambda'}\begin{bmatrix}l_3 & \frac{1}{2} & j_3\\ l_4 & \frac{1}{2} & j_4\\ \lambda' & S & J\end{bmatrix}\\ &\times\sum_{n'l'N'L'}M_{\lambda'}(n'l'N'L',n_3l_3n_4l_4)\\ &\times\sum_M A_R(N'L'NLM)\\ &\times\sum_m\Big[C^{\lambda(m+M)}_{lmLM}C^{\lambda'(m+M)}_{l'mL'M}V_r(n'l',nl,m)\\ &\times\sum_\mu(-1)^{1-\mu}C^{JM_J}_{\lambda(m+M)S\mu}C^{JM_J}_{\lambda'(m+M)S\mu}\Big]\end{aligned} \tag{13}$$

where $C^{LM}_{lml'm'}$ is the Clebsch-Gordan (C-G) coefficient $M_\lambda(nlNL,n_1l_1n_2l_2)$ is the Talmi transformation coefficient. The $A_R(N'L'NLM)$ and $V_r(n'l'nlm)$ are given as

$$A_R(N'L'NLM)=\Big\langle\phi_{N'L'M}(\sqrt{2}\vec{R})\Big|\Psi^*_{\pi^-}(\vec{R})\Psi_{\pi^+}(\vec{R})\Big|\phi_{NLM}(\sqrt{2}\vec{R})\Big\rangle, \tag{14}$$

$$\begin{aligned}V_r(n'l'nlm)\ \equiv&\ C\sqrt{(2l+1)(2l'+1)}(-1)^m\\ &\times\sum_{\lambda=0}^{\infty}i^\lambda f_{n'l'nl\lambda}C^{\lambda 0}_{l0l'0}C^{\lambda 0}_{l-ml'm},\end{aligned} \tag{15}$$

with

$$f_{n'l'nl\lambda}\equiv\Big\langle R_{n'l'}(\frac{r}{\sqrt{2}})\Big|j_\lambda(kr)H(r)\Big|R_{nl}(\frac{r}{\sqrt{2}})\Big\rangle. \tag{16}$$

where j_λ is the λ-order Bessel function, and R_{nl} is the radial wave function of the nucleon.

Since the excitation function at forward angle (about zero degree) of pionic DCX reaction is now a challenging topic with which the mode of short range correlation between nucleons may be identified, we investigate the contribution of the σ exchange to the excitation function of pionic DCX. On the other hand, the DCX reaction $^{14}C(\pi^+,\pi^-)^{14}O$ is, on the side of nuclear structure, the simplest DCX process and its angular distribution and excitation function have been discussed a lot[4, 5, 15, 16] in the conventional scheme without including the σ exchange. We then take the DCX process $^{14}C(\pi^+,\pi^-)^{14}O$ as an example to investigate the effect of the σ exchange on the excitation function at forward angle. For the forward angle (about 0°) process, $\vec{k_f} = \vec{k_i}$. $\vec{k_0}$ in the propagator of the σ meson (2) is then the on-shell momentum of the σ meson which holds the energy-momentum relation

$$\vec{k_0}^2 = \omega_\sigma^2 - m_\sigma^2, \quad (17)$$

where m_σ and ω_σ are the mass and energy of σ meson respectively. The value of ω_σ is decided by energy conservation in the process $\pi^+ n \to p\sigma$. Under the approximation of fixed scattering center, the energy of nucleon is approximately unchanged, i.e., $E_p \approx E_n$. Therefore, it is got from energy conservation that

$$\begin{aligned}\omega_\sigma &= \omega_\pi(k_i) + E_n - E_p \\ &\approx \omega_\pi(k_i). \end{aligned} \quad (18)$$

Substituting Eq.(18) into Eq.(17), the value of k_0 can be obtained

$$\begin{aligned}\vec{k_0}^2 &= \omega_\pi^2(k_i) - m_\sigma^2 \\ &= \vec{k_i}^2 + m_\pi^2 - m_\sigma^2. \end{aligned} \quad (19)$$

Because the incident energy of the pion is in the range $20 \sim 300$ MeV, i.e., $k_i = 20 \sim 300$ MeV and $m_\pi = 139$ MeV and $m_\sigma = 520$ MeV[17], we have $\vec{k_0}^2 < 0$. This means that σ exchange in the intermediate states is just a virtual process in contrast to the real process of π exchange. As well known, the destructive interference of isovector s- and p-waves in the πN-system causes the SCX cross section to undergo a deep minimum near $T_\pi \simeq 50$ MeV. From this point of view, one may speculate that σ exchange may play an important role in pionic DCX.

As in [9,15,16], we take the wave function of the ^{14}C as

$$\left|^{14}C(gs)\right\rangle = 0.404 \left|(0p_{3/2})^2 00\right\rangle + 0.915 \left|(0p_{1/2})^2 00\right\rangle. \quad (20)$$

Then we calculated the forward differential cross sections $\frac{d\sigma}{d\Omega}(0°)$ for the double analog transition process $^{14}C(\pi^+, \pi^-)^{14}O$ with incident pion energy in the range $k_i = 0 \sim 300$ MeV. In the calculation the coupling constants are taken as $\frac{g_{\pi NN}^2}{4\pi} = 14.4$ and $\frac{g_{\sigma NN}^2}{4\pi} = 7.303$ [17]. The calculated result and comparison with experimental data[18] are shown in Fig.1. The figure shows that the experimental excitation function of the DCX reaction ^{14}C(π^+, π^-)^{14}O in the range $T_\pi = 20 \sim 80$ MeV is reproduced very well. The contribution of the σ exchange mechanism is small in $\Delta(3,3)$ resonance region where the conventional sequential mechanism with the Δ resonances is dominant.

Comparing with the previous calculations, the present result of the excitation function of ^{14}C(π^+, π^-)^{14}O around $T_\pi = 50$ MeV agrees with experimental data at the same precision as that of the d′ resonance mechanism[3] and better than other conventional mechanisms[$4 - 6, 15, 16$]. It indicates that the σ exchange effect is very important in reproducing the excitation function of pionic DCX reaction at low energy.

In summary we have investigated the effect of the σ exchange on the excitation function of DCX reaction at low energy. The calculated result of the excitation function at forward angle of the DCX reaction ^{14}C(π^+, π^-)^{14}O in the energy range up to 80 MeV agrees well with the experimental data. The σ exchange mechanism reproduces the resonance like excitation function of low energy DCX reaction as well as the d' resonance mechanism.

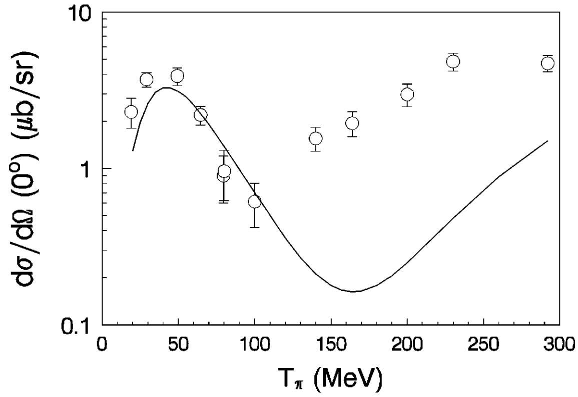

Fig. 1. Calculated result of the excitation function of the DCX reaction ^{14}C(π^+, π^-)^{14}O and the comparison with experimental data (taken from [18])

This work is supported by the National Natural Science Foundation of China and the Grant KJ951-A1-410 from the Academia Sinica. Helpful discussions with Professor H. C. Chiang are gratefully acknowledged. Zou thanks the support of the K.C.Wong Education Foundation, Hong Kong, for a visit to Beijing to start the collaboration. Liu thanks also the support of Peking University.

References

1. H. Clement, Prog. Nucl. Part. Phys. **29**, 175 (1992)
2. M. B. Johnson and C.L.Morris, Ann. Rev. Nucl. Part. Phys. **43**, 165 (1993)
3. R. Bilger, H. Clement and M. G. Schepkin, Phys. Rev. Lett. **71**, 42 (1993); **72**, 2972 (1994); K. Föhl *et al*, Phys. Rev. Lett. **79**, 3849 (1997)
4. M. A. Kargalis and M. B. Johnson, Phys. Rev. Lett. **73**, 38 (1994)
5. M. Kh. Khankhasayev, H. Sarafian, M. B. Johnson and Zh. B. Kurmanov, Phys. Rev. **C50**, 1424 (1994)
6. C.R.Ching, T.H.Ho and B.S.Zou, Nucl. Phys. **A510**, 670 (1990); **A513**, 697 (1990); A. Bobyk, A. Faessler and W. A. Kaminski, J. Phys. G 23, 375 (1997); Y. X. Liu, A. Faessler, J. Schwieger and A. Bobyk, J. Phys. **G 24**, 1135 (1998)
7. see for example, R. Machleidt, H. Holinde and C. Elster, Phys. Rep. **149**, 1 (1987); A. Macheleidt, Adv. Nuc. Phys. **19**, 189 (1989)
8. T.S.H. Lee and D.O. Riska, Phys. Rev. Lett. **70**, 2237 (1993)
9. M. B. Johnson, E. R. Siciliano and H. Sarafian, Phys. Lett. **B 243**, 18 (1990)

10. B. S. Zou and D. V. Bugg, Phys. Rev. **D 48**, 3948 (1993); **D 50**, 591 (1994); D. V. Bugg, A. V. Sarantsev and B. S. Zou, Nucl. Phys. **B 471**, 59 (1996); V.V.Anisovich and A.V.Sarantsev, Phys. Lett. **B382**, 429 (1996)
11. Particle Data Group, Phys. Rev. **D 54**, 1 (1996) and references therein
12. G. A. Miller, Phys. Rev. Lett. **53**, 2008 (1984); Phys. Rev. **C 354**, 327 (1987)
13. H. C. Chiang and B. S. Zou, Nucl. Phys. **A 496**, 729 (1989)
14. M. B. Johnson and D. J. Ernst, Phys. Rev. **C 27**, 709 (1983)
15. M. Blezynski and R. Glauber, Phys. Rev. **C 36**, 681 (1987)
16. C. R. Ching, T. H. Ho, B. S. Zou and H. C. Chiang, Phys. Lett. **B 252**, 192 (1990)
17. B. D. Serot and J. D. Walacka, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum Press, New York, 1986), Vol. 16
18. M.J. Leitch *et al.*, Phys. Rev. Lett. **54**, 1482 (1985); M.J. Leitch *et al.*, Phys. Rev. **C 39**, 2356 (1989)